USING THE BLACK-SCHOLES MODEL TO PRICE CHAPTER 11 SECURITIES:

An Illustration

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AUTHOR'S NOTE

This document, along with its sister, "Pricing The Securities of Companies in Chapter 11: *An Options Approach*," are reissues of papers that I published privately in 1990.

When I decided to make these papers available in electronic form (i.e., as Adobe® Acrobat® files), it should not be surprising that I encountered some technical challenges in working with seventeen year-old file formats. The original documents were written in WordPerfect®, and the graphs were generated in Lotus® 1-2-3®. While the text portion of the documents required only minor reformatting, the graphics from the original file were a complete mess.

The graphs in this reissue have been rebuilt completely from scratch. In my 1990 efforts I had built fairly complex tables of data in order to apply the Black-Scholes formulas, and their first derivatives, to the underlying (though 'mock') data on Belly-Up, Inc. For this, 2007 re-publication I used Visual Basic® to write custom functions in Microsoft Excel® to re-generate the graphs. To my relief, the resulting graphs in all cases match those in my original publications.

Overall, the end result are two papers virtually unchanged from their original versions. Some slight reformatting has been necessary, and I did (of course) find (and correct) one typo.

However, while in the process of republishing these papers I have also made the following deliberate changes: First, for this paper (but not for its sister) I have added a *Table of Contents*, a *Table of Figures*, and, of course, this *Author's Note*. Second, I also changed "one" phrase; in my 1990 papers I frequently spoke about "the value of the company" (or I used phrases that are close cousins) in discussing my hypothetical Belly-Up, Inc. For the current incarnations of my papers I have changed that wording to "the value of the company's assets" (or appropriate close cousins). To me, this "one" change makes a distinction – about the separate process and importance of valuing a company's underlying assets before an investor can begin to value that company's securities - that should have been clearer in my original papers.

William H. White
February 2007

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INTRODUCTION

This paper will illustrate, through the use of a hypothetical case, how the Black-Scholes model can be employed to value the securities of companies in Chapter 11 proceedings.[1] We begin with the perspective that Chapter 11 securities are comprised of call options on company assets. To the extent that this view is correct, it means that any variable which affects option value will in turn influence Chapter 11 security prices. Further, since the price behavior of options is often markedly non-linear, it becomes imperative to work with first derivatives of the Black-Scholes model in our analysis. These first derivatives will be used to show the sensitivities of debt and equity prices to changes in underlying option variables.[2]

This paper is by and large a supplement to our previous effort, "Pricing the Securities of Companies in Chapter 11: An Options Approach." In that article we argued the following descriptions of Chapter 11 securities:

- *Owning senior debt is equivalent to having written a covered call on company assets. Specifically, it is equivalent to 1) being long the assets of the company, and 2) being short a European call option on the assets, where the strike price equals the full, face-value of the senior debt.*

- *Owning junior debt is equivalent to holding a bullish vertical spread position on company assets. Specifically, it is equivalent to 1) holding a European call option on those assets, with a strike price equal to the full, face-value of all senior debt, and 2) having written a European call option on company assets, with a strike price equal to the total face values of both senior and junior debt.*

- *Owning common stock is properly viewed as equivalent to holding a European call option on company assets, with a strike price equal to the full, face-value of all outstanding debt.*

Further, for all of these options, expiration occurs at the end of bankruptcy proceedings, and any uncertainty about final settlement amounts is treated like a 'negative dividend'.

[1]The author wishes to thank Edward I. Altman, Max L. Heine Professor of Finance at the New York University Stern School of Business, for his encouragement and editorial suggestions on this paper, as well as Sarah M. Treu, Vice President, The Bank of New York, and Yakil Polak, Distressed Securities Analyst, Gruss & Co., for their insights into the bankruptcy process.

[2]The main, and virtual sole, source for option valuation formulas and general observations on options, is John C. Cox and Mark Rubinstein, *Options Markets* (Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1985).

ASSUMPTIONS

As we fully discussed in our previous paper, there are complications in applying options theory to Chapter 11 securities. In particular, the Black-Scholes model was developed for listed options, where strike prices and time to expiration are known and fixed. With Chapter 11, however, investors face uncertainty about both the duration of bankruptcy proceedings (i.e., time to expiration), and the amount of their claims that courts would honor in an enforced settlement (i.e., strike prices).

Despite these, and other, real-world complications, we can best serve the purposes of this paper by keeping our calculations and explanations as uncluttered as possible. Accordingly, we will depend on a number of assumptions when calculating security prices. Although these assumptions will make our results somewhat simplistic, we feel that the usefulness of our approach under real-world conditions should still be quite evident.

First, we repeat those assumptions introduced at the beginning of our prior paper:

- There are only 3 classes of claims: senior debt, junior debt, and equity.
- All claims against the firm are in the form of publicly-traded securities.
- None of the debt claims have specific assets pledged as collateral.
- The company successfully suspends all interest payments (and any other non-operating cash disbursements) during the course of Chapter 11 proceedings.
- Claimants are indifferent to the form of payment in settlement of their claims.

Then, except in instances where we specifically state otherwise, our calculations assume that:

- There are no taxes or transaction costs, and markets are perfectly liquid.
- All assumptions needed to establish the general validity of the Black-Scholes model hold true.
- In the event of a 'cram-down' (a court-imposed settlement):

    1) strict and full priority of claims will be honored by the bankruptcy court.

    2) the court will not award any post-petition interest.

- The ultimate settlement date is known with certainty, with all claims paid on that day.

The net effect of these assumptions is that they allow us to find the 5 variables needed for Black-Scholes valuations without any significant complications.

BELLY-UP, INC.

Although we are dealing with a hypothetical case, let us inject some realism into the discussion and set up our company - Belly-Up, Inc. - as 'the' average firm in Chapter 11. To do this we say that Belly-Up has $400 million in total liabilities,[3] of which $220 million is senior debt, and $180 million is subordinated debt.[4] The current value of the firm's assets is $360 million,[5] and that value has an annual standard deviation of .28.[6] We will assume that bankruptcy proceedings will take 27 months,[7] and we will use an interest rate of 7.75%.[8]

It is not important for our description of Belly-Up, Inc. to be a precise representation of 'the' average firm in Chapter 11. These values are picked only to give us a reasonable starting point for our discussion. In fact, we will now proceed to replace each one of our initial, 'average' assumptions with a wide range of alternate values. Through studying the effect of these new inputs on final security prices we expect to learn a lot about what 'drives' the value of Chapter 11 securities.

At any rate, in looking for a staring point for our discussion we have covered the necessary variables:

S:	$360.0 million	t:	2.25 years (27 months)
K_{senior}:	$220.0 million	r:	7.75%
K_{junior}:	$400.0 million ($180 + $220 million)	σ:	28.0%

[3]See Dun & Bradstreet, Business Failure Record, 1988 (preliminary). The average liability per failure in early 1980s was approximately $400 million. Figures from that time period were taken to maintain consistency with data taken from work by Altman.

[4]See Edward I. Altman, *Corporate Financial Distress* (New York: John Wiley & Sons, 1983). Short-term debt as percentage of total debt for all nonfinancial corporations was approaching 45% in 1982 (Chart 1-6, p.45). We presume that all short-term debt, plus some of the long-term debt is senior. As a best guess, we are setting senior debt slightly larger than junior debt.

[5]In developing his Z-score model, Altman found that one year before bankruptcy the market value of equity (including preferred) of those firms he studied to be 40% of book value of liabilities (p. 109). However, we cannot help but suppose that the total value of a firm when entering Chapter 11 has typically fallen below the face value of debt, and have arbitrarily set Belly-Up's market value 10% below book value of debt.

[6]See Eugene F. Brigham, *Financial Management* (New York: The Dryden Press, 1985). Brigham states that standard deviation of an average stock in late '70s/early '80s was 28% (p. 195).

[7]See Altman, p. 26.

[8]The interest rate used as an input for the Black-Scholes model should be an appropriate risk-free rate. Accordingly, when initial calculations were performed (December, 1989), we used the prevailing yield-to-maturity on 27-month Treasury notes.

Belly-Up, Inc. (cont'd)

The following table shows our options definition of each security, and then the value for each security as derived with the Black-Scholes model. Market values in this table are also expressed as Trading Values, which is a bond's price expressed as 'cents on the dollar'. Finally, to provide some perspective, we show the maximum (i.e., risk-free) value that bonds can reach. This maximum value is simply the price that bonds would take if they were zero-coupon treasury notes (the value of 'risk-free' equity is a plug):

Security	Options Equivalence	Our B-S Valuation (millions)	Trading Value	Risk-Free Value: PV @ 7.75% (millions)
Senior debt	long assets, short 220 call	$183.3	83¢	$186.0
Junior debt	long 220 call, short 400 call	$107.1	59¢	$152.2
Equity	long 400 call	$69.6	---	$21.8
TOTALS:		$360.0		$360.0

Since asset values at Belly-Up, Inc. are far in excess of what is needed to satisfy senior claimants, it is not surprising to find that the Black-Scholes model prices senior debt very close to its risk-free value. Junior debt, however, is another story. Belly-Up, Inc.'s $360.0 million in assets should be sufficient to repay all debtholders, at least in present value terms, but there clearly is not the cushion for junior debt that there is for senior debt. In fact, the Black-Scholes model prices Belly-Up's junior debt far below its risk-free value. Obviously the model is taking account of the significant risk that asset values may drop below what is needed to repay junior debt.

Finally, we turn to common stock. Under the risk-free scenario, we plug a value of $21.8 million for Belly-Up's equity, which is properly interpreted as the minimum value of Belly-Up's common. However, our Black-Scholes valuation of equity, $69.6 million, is more than triple this risk-free amount. Clearly, our use of the Black-Scholes model reflects the possibility that Belly-Up's asset values may grow. Should that occur, any increase beyond $400.0 million will accrue entirely to the company's common stockholders.

One last item worth noting is that in conducting our analysis of Belly-Up, Inc. we want to pay particular attention to the behavior of junior debt. Though we have assumed that there are only 3 classes of claims against a firm, in reality there are many more. However,

> *all of the intermediate claims against a company in Chapter 11 can be described as taking a bullish vertical spread position with call options on the company.*

Thus, the price behavior of the subordinated debt in our model will be the best indicator of what influences the value of most claims against a bankrupt firm.

Interest Rates

In establishing prices for the securities of Belly-Up, Inc. we assumed an annual interest rate of 7.75%. As we can see in Figure 1, the market value of senior debt falls as interest rates increase, and this loss essentially accrues to the common stockholders. These price patterns hold true because bankruptcy settlements will be paid with 'cheaper' dollars as rates rise. Belly-Up's junior debt, however, is virtually immune to interest rate risk. This implies that as interest rates increase the losses experienced by junior

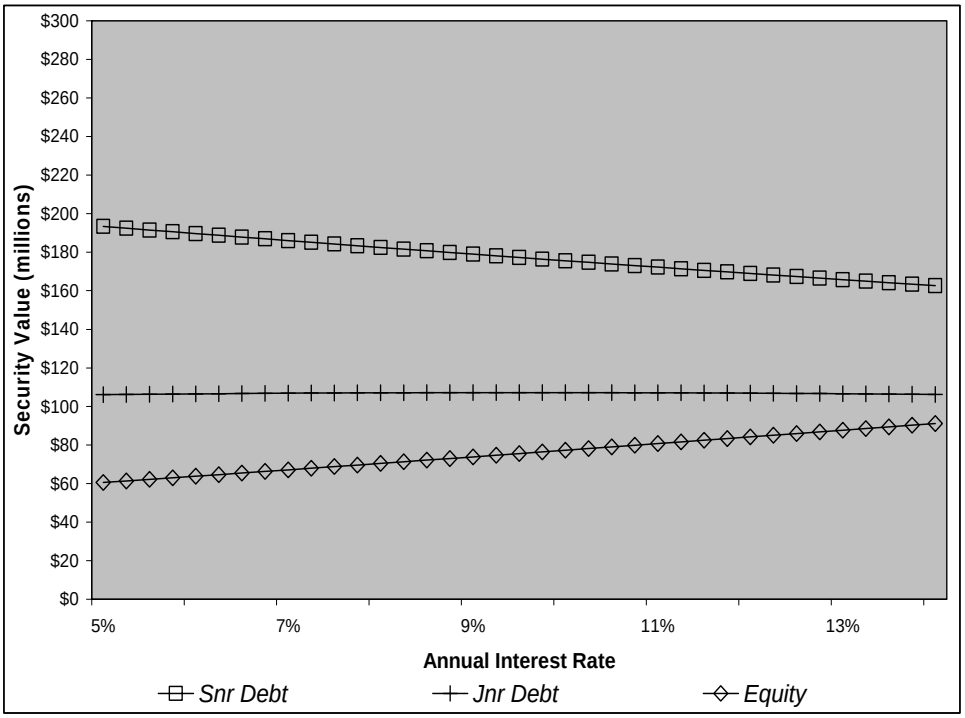

Figure 1 - Security value as a function of interest rates

debtholders in having their claims paid with cheaper dollars are almost exactly offset by the gains in being able to pay senior creditors with cheaper dollars.

Interest Rates (cont'd)

Another way to understand the effect of interest rates is to translate Figure 1 into a rate-of-return diagram, as is done in Figure 2. An investor might use Figure 1 as a tool for pricing Belly-Up's securities under various initial interest rate assumptions. Figure 2 represents the returns and losses an investor faces if she buys Belly-Up's securities when interest rates are 7.75%, and then interest rates change.

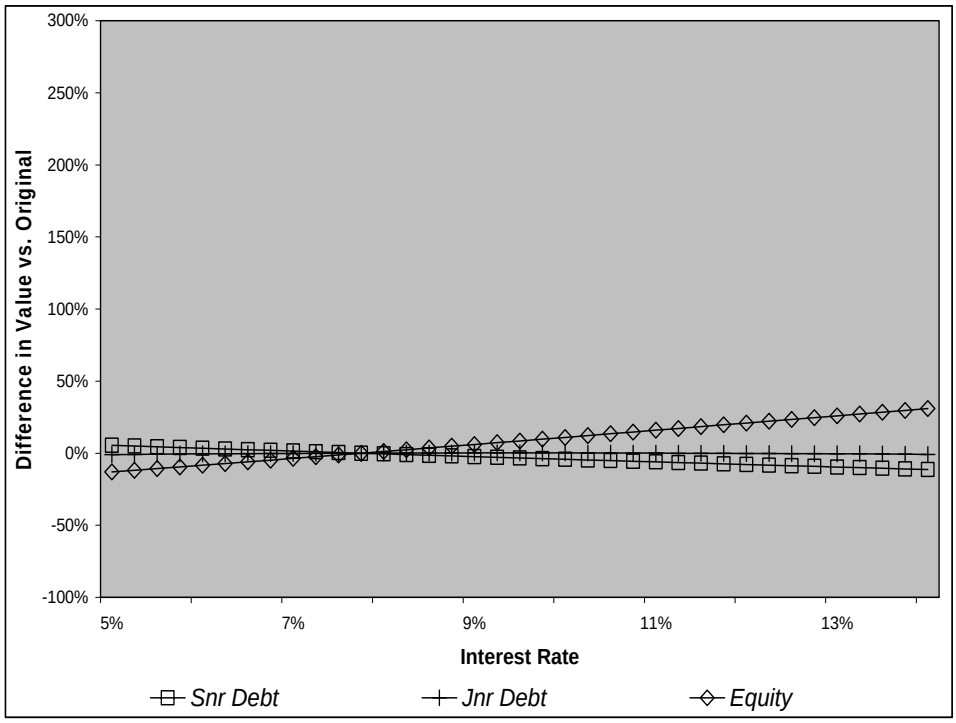

Figure 2 - Returns realized as interest rates change from 7.75%

Interest Rates (cont'd)

Having seen the effect that interest rates have upon the prices Belly-Up securities, we want to know whether our conclusions can be extended to Chapter 11 securities in general. Of particular concern is that the results seen in Figures 1 and 2 depend on the expected length of bankruptcy proceedings (among other assumptions).

To test this dependency, we take the first derivative of the Black-Scholes formula against changes in interest rates, and then make appropriate adjustments for the fact that junior debt represents a portfolio of two calls, and that senior debt is akin to a covered call.[9] The results are shown in Figure 3. What we find most notable is that the interest rate sensitivity of

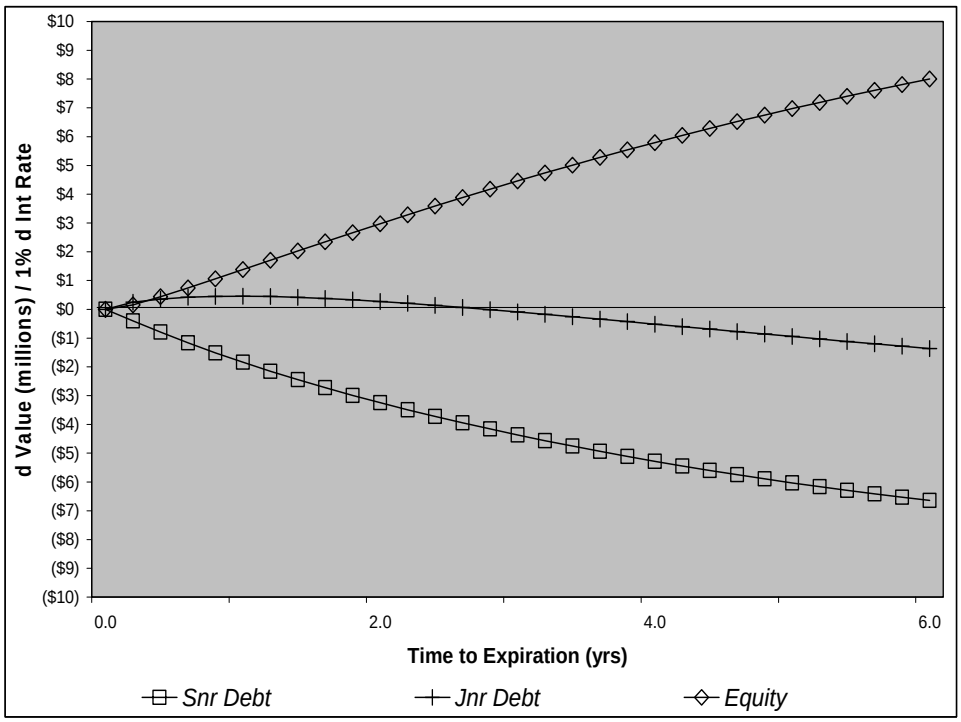

Figure 3 - Security price sensitivity, over time to expiration, to change of 100 basis points in interest rates

junior debt is virtually unaffected by changes in the expected length of bankruptcy proceedings.

[9]See Cox and Rubinstein, p. 221, for first derivatives of the Black-Scholes model.

Volatility

In establishing our initial values for the securities of Belly-Up, Inc., we assumed that the annual standard deviation in the value of company's assets is .28. The security values under different assumptions about volatility are shown in Figure 4. As can be seen, Belly-Up's senior debt is not greatly affected by the volatility of the value of the

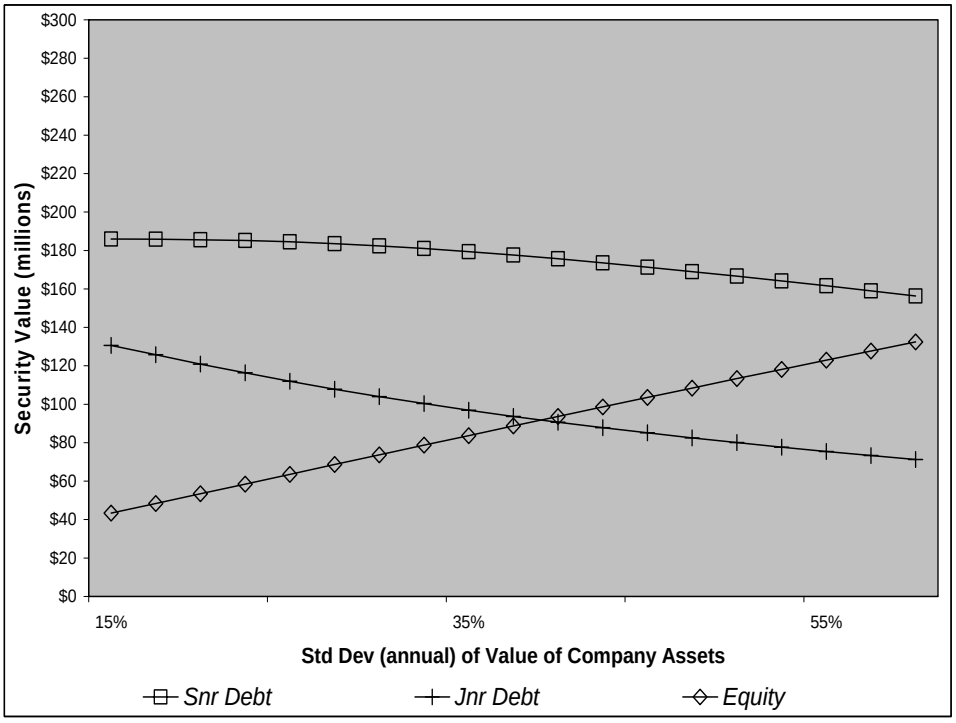

Figure 4 - Security value as a function of annual standard deviation of value of company's assets

company's assets. This result is so because the call embedded in senior debt is fairly deep in-the-money, making the value of that call relatively stable.

The impact of volatility is much more pronounced on junior debt and equity. Figure 4 shows that the value of equity claims increases with the volatility of asset valuation. This happens because common stock of a Chapter 11 firm is equivalent to being long a call on that firm, and call values have a positive relationship with volatility. The junior debt of Belly-Up, Inc., however, has virtually nothing to gain and much to lose with higher volatility. This is because the purchased call underlying junior debt is already well in-the-money, leaving that call somewhat stable. However, the underlying written call (owned by the equity claimants) is basically at-the-money, so gains realized by stockholders with increased volatility come largely at the expense of junior debtholders.

Volatility (cont'd)

Again, it is worthwhile to translate our initial-value diagram (Figure 4) into a rate-of-return diagram (Figure 5). In Figure 4 we saw that as volatility increases, there is roughly a $1 gain in the value of equity for every $1 loss in the value of junior debt. Yet, since the initial market value of equity ($69.6 million) is so much lower

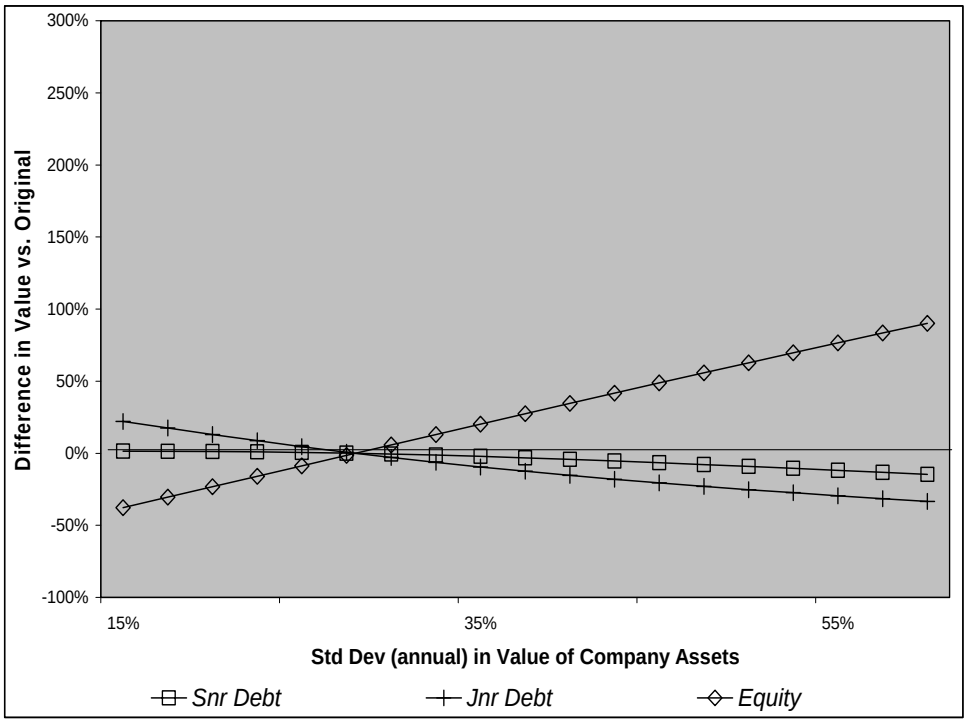

Figure 5 - Returns realized as annual standard deviation of company asset value changes from .28

than the initial market value of junior debt ($107.1 million), those dollar trade-offs translate into much higher rates-of-return (and loss) for equity than for junior debt.

Volatility (cont'd)

Having made the above observations about how the volatility of the value Belly-Up's assets affects its security prices, we want to get a feel for how much our diagrams and conclusions depend on our assumptions about other variables. In particular, we suspect that the 'sigma sensitivity' of Belly-Up's securities are affected by the expected time to settlement, and by the underlying company asset values themselves.

The length of bankruptcy proceedings, as we see in Figure 6, does affect the sensitivity of

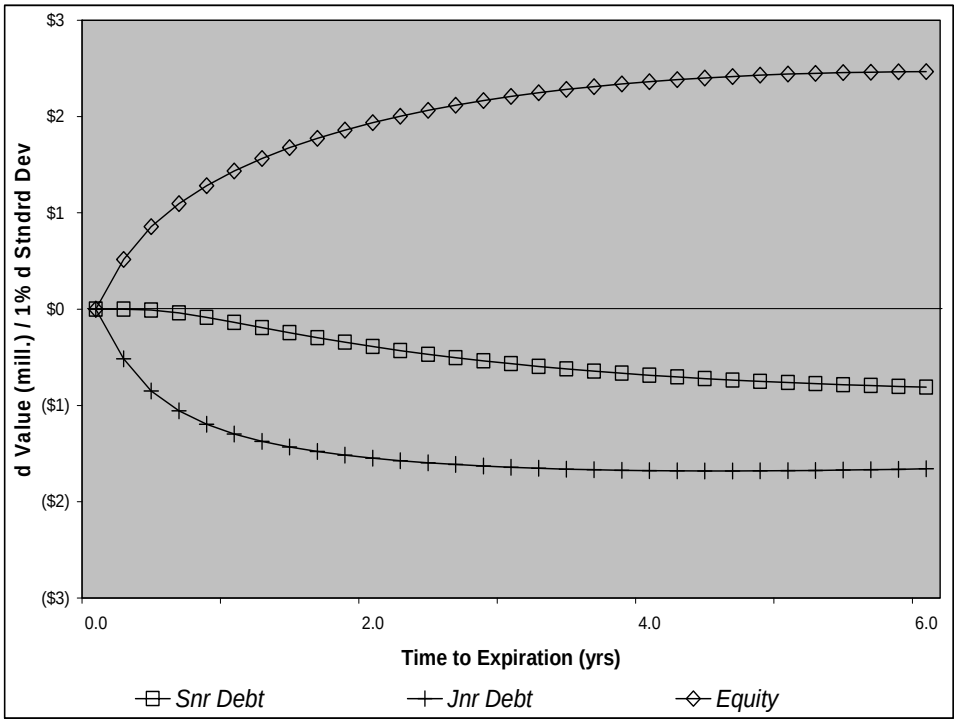

Figure 6 - Security price sensitivity, over time, to 1 point change (i.e., .28 to .29) in annual standard deviation of company asset values

Belly-Up's security prices to changes in volatility, although 'sigma sensitivity' tends to reach its extremes with more than 2 to 3 years remaining to settlement.

Volatility (cont')

The effect of asset values themselves on 'sigma sensitivity', shown in Figure 7, is much more interesting. Clearly, what we see is that security prices are most sensitive to changes in volatility when those securities are at-the-money. As the options upon which our securities are modeled become either deep-in- or deep-out-of-the-money, their 'sigma sensitivity' lessens.

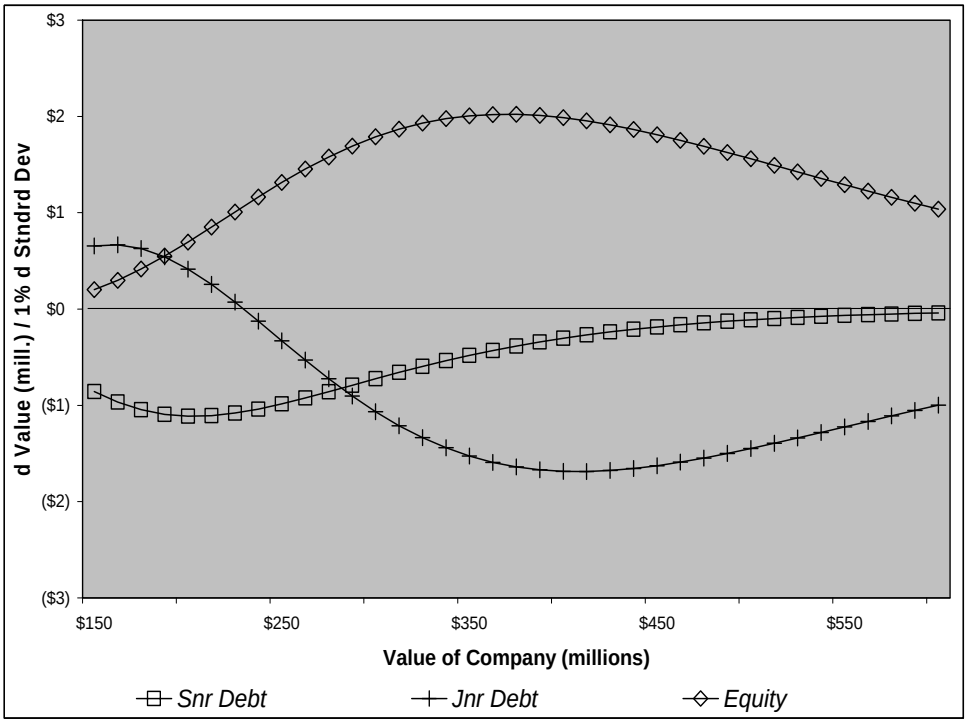

Figure 7 - Security price sensitivity, as a function of value of the company's assets, to a 1 point change in the annual standard deviation of those asset values

Length of Time in Chapter 11

We move now to a variable that strongly affects the value of each of the securities of Belly-Up, Inc. We initially assumed that reorganization will take 27 months, which Altman found to be the mean length of reorganization for companies in Chapter 11. Security prices for Belly-Up under different assumptions about the length of Chapter 11 proceedings are shown in Figure 8.

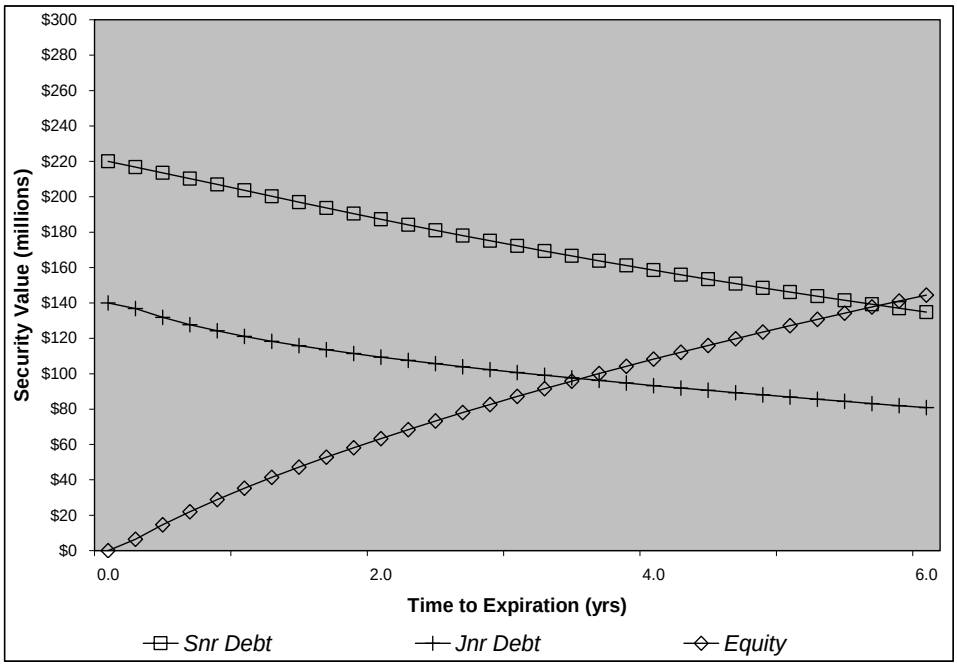

Figure 8 - Security value as a function of time to settlement

One aspect of Figure 8 we find particularly noteworthy is how well it illustrates the conflicting interests of creditors and equityholders. While bondholders clearly want to push for the quickest possible resolution of Chapter 11 proceedings, stockholders have a very strong interest in delaying such proceedings for as long as possible. When we remember our options explanation for valuing Chapter 11 equity, the reasons for stockholders' recalcitrance become evident. Longer proceedings allow stockholders to 'pay off' debtholders with ever-cheaper dollars (assuming no post-petition interest). A longer time to expiration also benefits stockholders because it puts volatility to work for a longer period of time, thus increasing the chance that the stock (i.e., the call) will finish in-the-money.

Time in Chapter 11 (cont'd)

As we have done with our other variables, let us translate our price level diagram (Figure 8) into a rate-of-return diagram (Figure 9). Figure 8 can be thought of as a diagram showing the initial prices of Belly-Up's securities under various expected lengths of bankruptcy

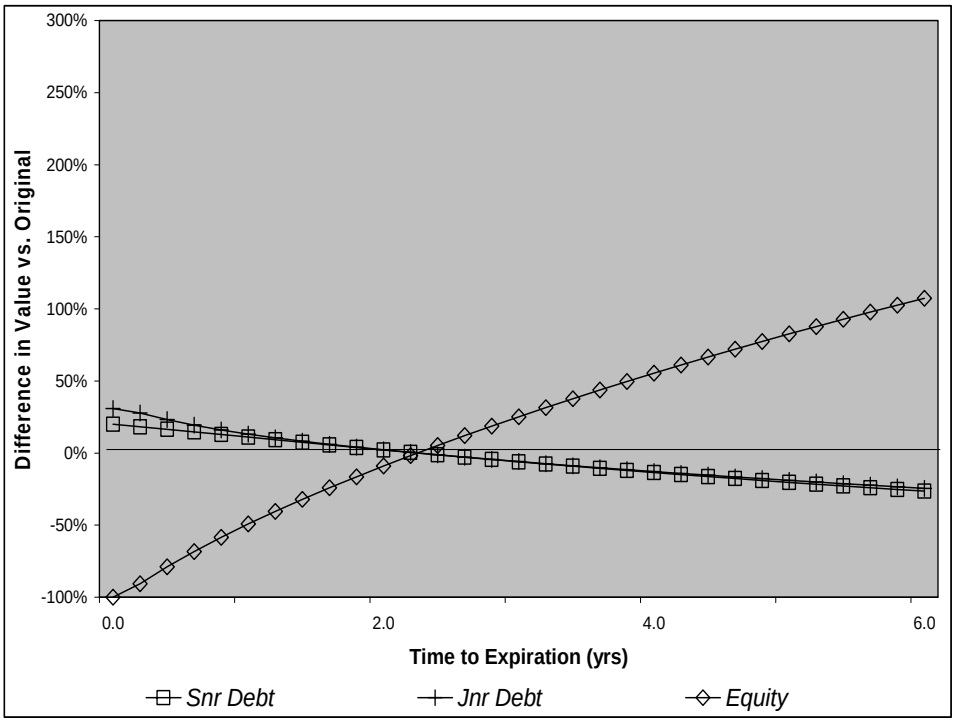

Figure 9 - Returns realized as time to expiration changes from 29 months

proceedings. Figure 9 shows what happens when expectations about the duration of Chapter 11 proceedings change. What we find notable in Figure 9 are that when the time to settlement changes, the rates of return realized by both senior and subordinated debtholders are almost identical, whereas the rates of returns recognized by Belly-Up's stockholders are much greater.

Time in Chapter 11 (cont'd)

Since the expected time to settlement of claims has such a large influence on the prices of Belly-Up's securities, we want to understand fully how this influence works. A critical step for us then, especially if we want to draw general inferences about Chapter 11 securities from the specific case of Belly-Up, is to analyze security price sensitivities to changes in time to settlement. Our measure for this is theta, which is defined as -*d*(security)/*d*(time to settlement).[10] Note that our graphs divide theta by 12 to show security price sensitivity to 1 month changes in time to expiration.

Our first look at theta is shown in Figure 10, which depicts security theta over varying projected lengths of bankruptcy. As can be seen, all securities have a fairly stable theta with roughly more than 2 years to settlement. As is typical with near-the-money options (i.e.,

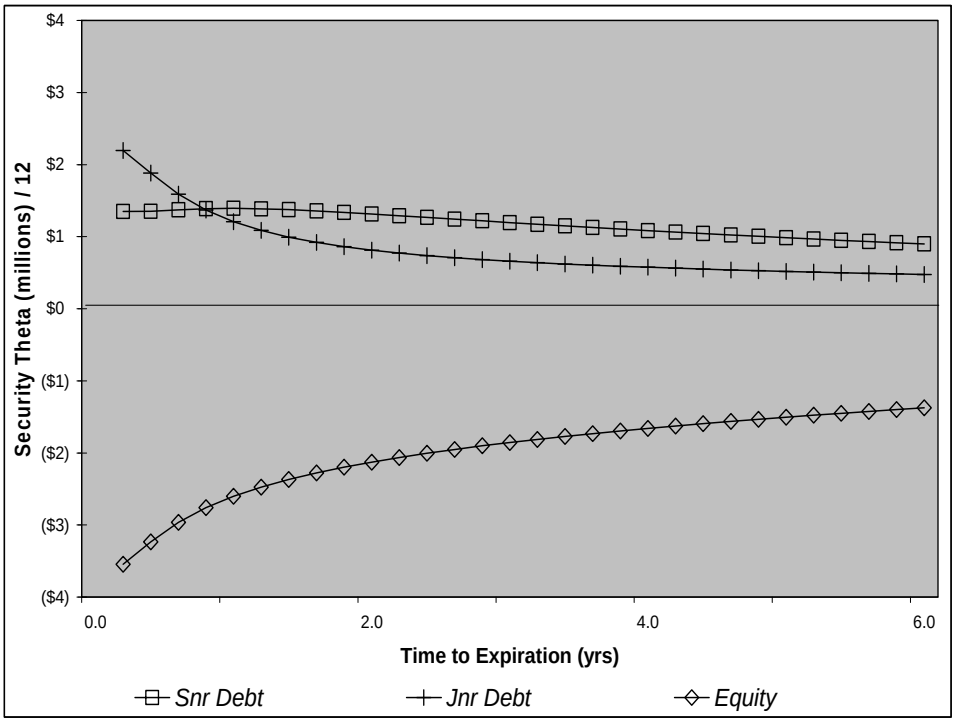

Figure 10 - Security theta as a function of time to settlement

junior debt and equity) theta magnifies as the time to expiration (settlement) draws very close. It is also worth noting that what we see with senior debt and equity in Figure 10 will be true in all Chapter 11 proceedings - *ceteris paribus*, senior debt will always increase in value and equity will always decrease in value with the passage of time.

[10]Theta reverses the sign of this first derivative in order to show option sensitivity to the passage of time (i.e., to decreases in time to expiration).

Time in Chapter 11 (cont'd)

Perhaps the most interesting influence upon theta is the value of the assets of the company itself (shown in Figure 11). We see that thetas of all securities reach their extremes once the underlying value of company assets rises above a certain point. As the value of those assets

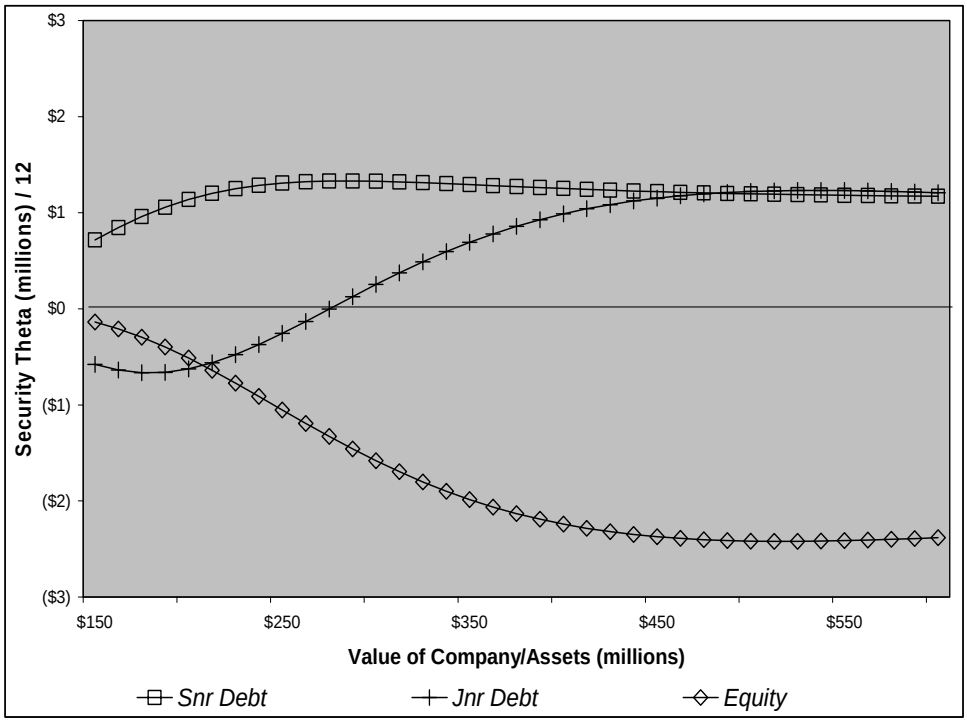

Figure 11 - Security theta as a function of company value

fall, though, so does the sensitivity of Belly-Up security prices to the passage of time.

The moderation in thetas as asset valuation decreases is largely accounted for by the fact that the dollar value (i.e., price) of all securities, and therefore the dollar level of their sensitivities, will fall with a drop in the value of company assets. In practice, it might well be more useful to express all of our sensitivity graphs as elasticity graphs - that is, to divide the various first derivatives in these graphs by the security prices. However, in the interest of brevity, we will not pursue these calculations.

Company Value

When we established initial price levels for the securities of Belly-Up, Inc., we assigned a value of $360 million to the company's assets. Since this number was selected rather arbitrarily, we now want to look closely at other numbers and their effect on Belly-Up's security prices. Figure 12 shows exactly that. As can be seen,

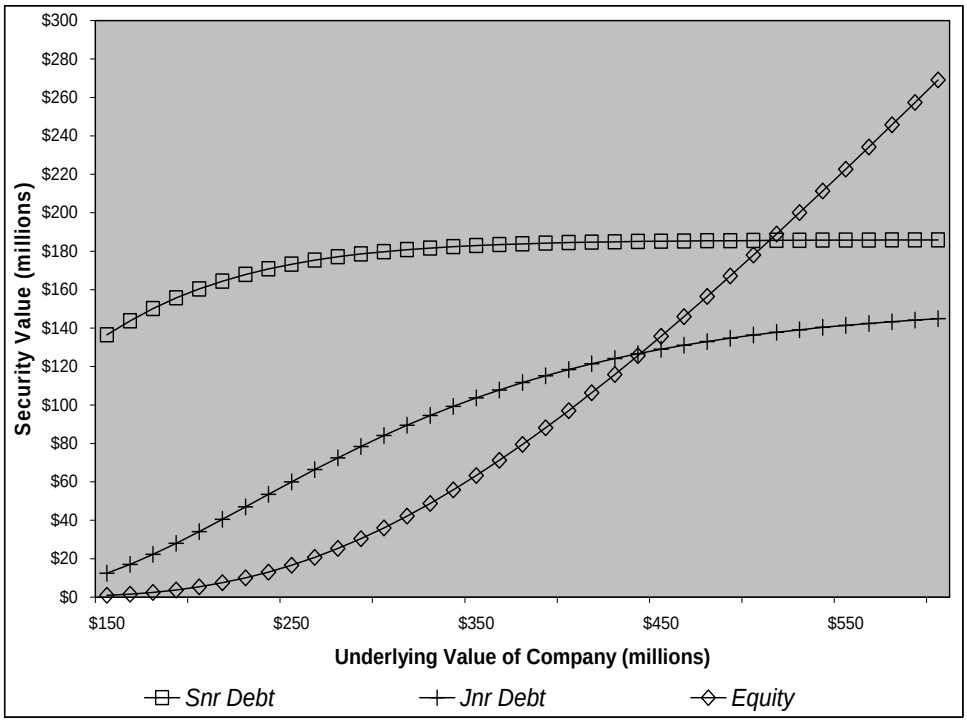

Figure 12 - Security value as a function of company (assets) value

we have plugged in a wide range of assumptions about the underlying asset value of Belly-Up, Inc. - from a low of $150 million to a high of $600 million.

What strikes us about Figure 12 is how clearly it illustrates our options approach to valuing the securities of a company in Chapter 11; these security prices (by virtue of how we defined the securities) are typical option pay-off diagrams. The price pattern for Belly-Up's senior debt shows how we have constructed that security as a covered, written call (this also happens to be a synthetic put). The price behavior of Belly-Up's common stock illustrates our definition of equity as a call option. The diagram for junior debt is that of a bullish vertical spread.

Company Value (cont'd)

What Figure 12 also highlights is the *importance* of asset values in establishing security prices for Belly-Up. To make this point even more dramatically, we again follow our practice of translating a price-level diagram into a rate-of-return diagram (Figure 13). As the company value (i.e., the value of its assets) changes from $360 million the rates of return realized by Belly-Up's common stockholders

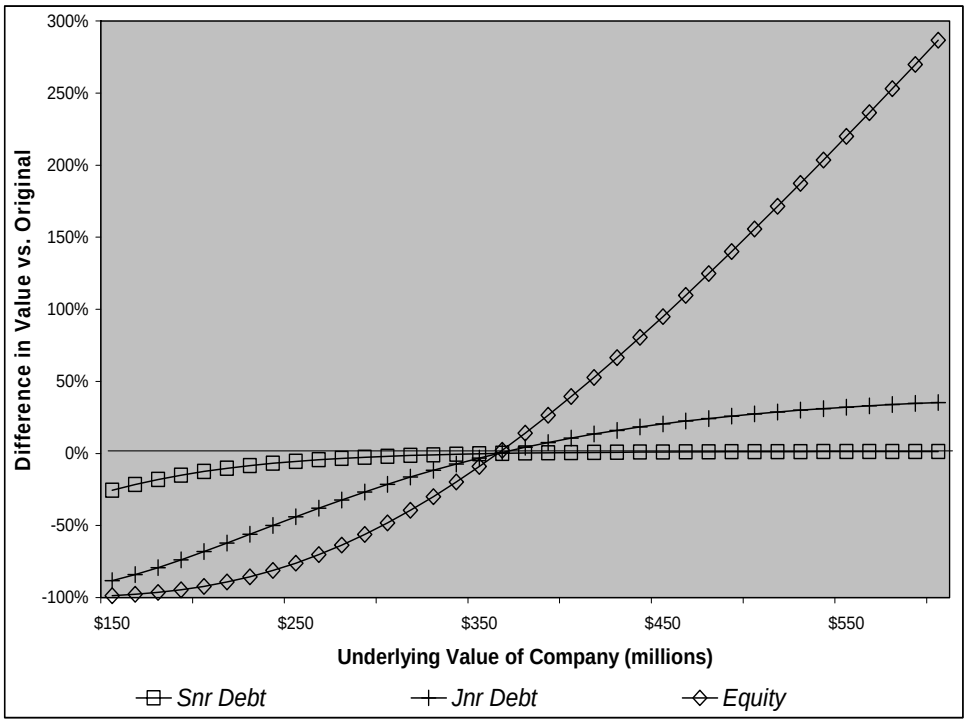

Figure 13 - Returns realized as company value (i.e., the value of its assets) changes from $360 million

are eye-popping. But this, of course, is the case when holding any near-the-money option. We note that junior debt shows potential losses of up to 88%, but potential gains of only 35%.

Senior debt is intriguing. Many investors in Chapter 11 securities believe in taking 'conservative' positions - which, to them, means buying senior debt only. Yet our contention is that senior debt of a Chapter 11 company is analogous to a covered call on the firm's assets, which in turn is analogous to having written a synthetic put on those assets. Not many people regard put-writing as a conservative investment strategy, and Figure 13 shows that while senior debt looses the least when asset values fall, senior debt (at least when it is well in-the-money) can also be a classic no-win bet.

Company Value (cont'd)

Given the importance of company value in determining Belly-Up's security prices, we are very concerned with security price sensitivity to changes in asset values. This measure, *d*(security) / *d*(Belly-Up assets), is delta (options traders will recognize this as the hedge ratio of an option). In looking at influences on delta, we again focus on time to settlement and the underlying value of company assets. (We did examine the effect of changes in the volatility of asset valuation, but, somewhat surprisingly, found this to have little impact on deltas.)

Figure 14 shows security delta as a function of time to settlement. As is the case with any call option, delta approaches zero or one as t approaches zero. This result simply means that

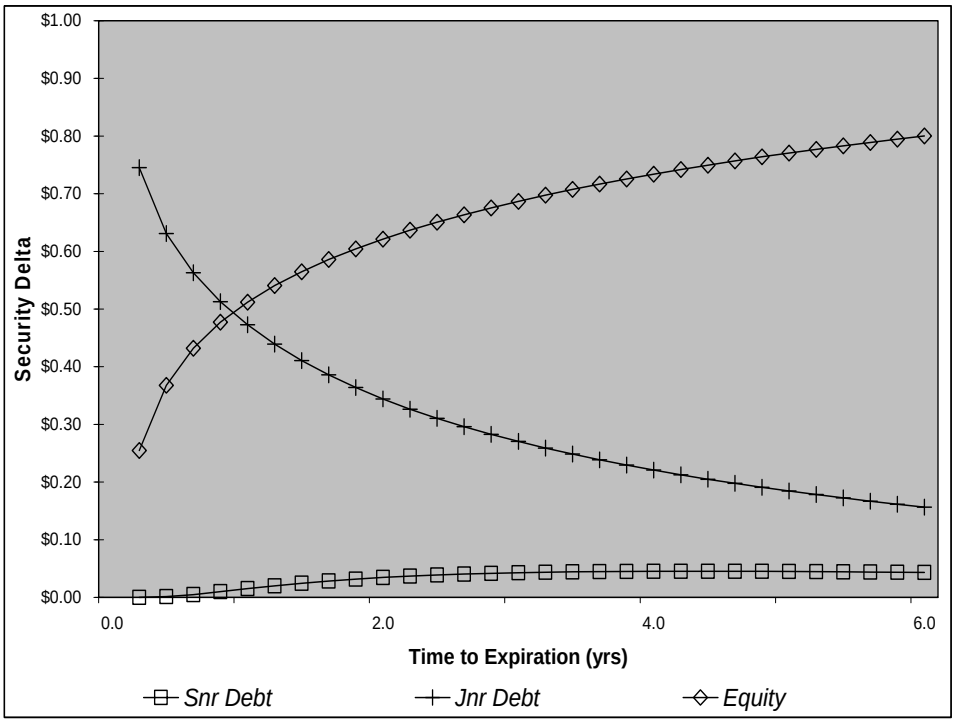

Figure 14 - Security delta as a function of time to settlement

at expiration a call option is either in-the-money or out-of-the-money.

A change in asset values, shown in Figure 15, has rather profound influences on security delta. The deltas of both senior debt and equity should be readily understandable, especially if we remember that senior debt is analogous to a synthetic put. The impact of the company's underlying asset values upon the delta of junior debt is

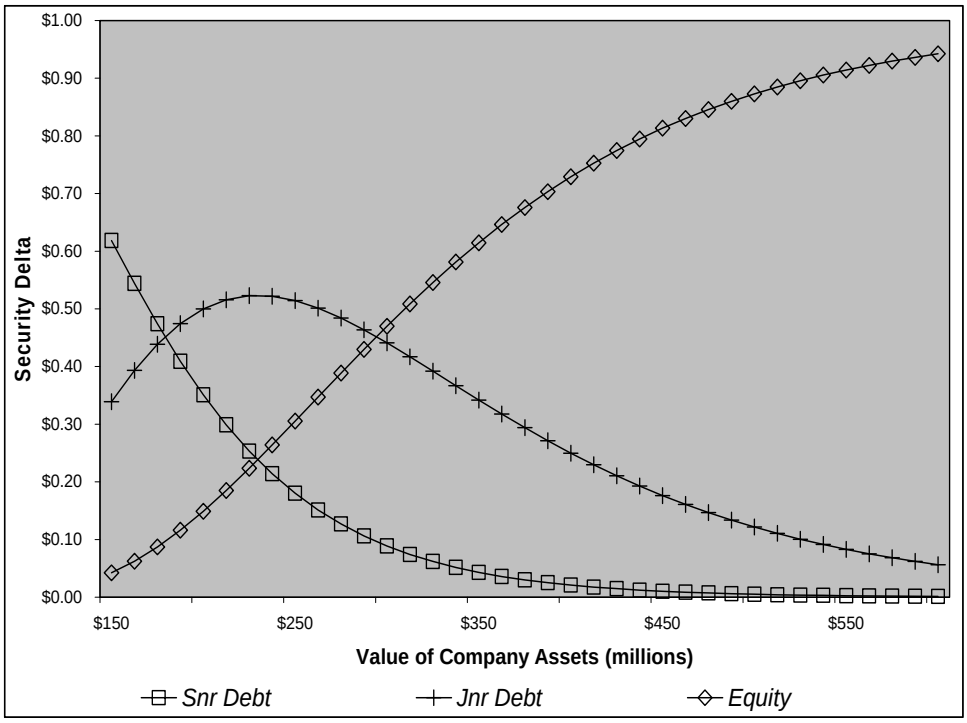

Figure 15 - Security delta as a function of company (i.e., asset) value

more complex, though we note that this delta peaks at the present value ($186.0 million) of the senior debt. With our options model, that face value of senior debt is the strike price of junior debt's underlying purchased call.

Before leaving this subject, it is worth noting that deltas are non-negative for each of Belly-Up's securities - meaning that an increase in the value of a Chapter 11 company can never lessen the value of any one of its securities. Obviously equity holders, who are long a call, and junior debtholders, who own a bullish vertical spread on the company, only stand to gain as the value of the company gains. Senior debtholders have *written* a call (to the junior debtholders), and will suffer losses on that option as company asset values rise. However, senior debtholders are also long the company itself, so their option losses are always more than covered.

Settlements to Debtholders

The final variable which we will consider in examining our Chapter 11 pricing model is the face value of claims against Belly-Up, Inc. One approach we could take here is to examine the security prices for Belly-Up, Inc. under capital structures other than our current assumption of $220 million in senior debt and $180 million in junior debt.

A more interesting analysis, though, and the one which we will follow, is to view what happens to Belly-Up's security prices when it is expected that strict and full priority of claims will not be honored. Such reductions in the face value of debt typically occur through negotiations, where creditors voluntarily waive a portion of their claims in order to expedite final settlement. These reductions also effectively occur in a cram-down if the bankruptcy court pays settlements with over-valued securities.

In our prior paper we argued that an investor using our approach should price Chapter 11 debt by treating any expected reduction in the legally recognizable value of claims like an at-expiration, 'negative dividend'. We then went on to review the Cox and Rubinstein approach to dividends in some detail. Briefly, Cox and Rubinstein contend that an investor must revert to a binomial options pricing model to account for dividends, and must also express expected dividends as a function of the underlying stock price.

While the Cox and Rubinstein approach is the 'correct' way of treating dividends, in practice most investors account for dividends simply by subtracting the present value of expected dividends from the current value of the stock. By extension, this means that one practical way to treat expected reductions in settlement amounts with Chapter 11 securities would be to add the present value of expected reductions to *S*, the value of company assets (we add this amount because we are subtracting a 'negative dividend').

There is, however, another way to account for expected reductions in recognizable claim amounts, and this method is somewhat simpler than making adjustments to asset values. Since reductions in the recognizable value of debt occur at-expiration, we could show, through a brief examination of option pay-off equations, that an investor can subtract that expected diminution in claims directly from *K*, which is the full face value of those claims. Further, since the Black-Scholes model converts strike prices to their present value, this approach would achieve the necessary discounting of our 'negative dividend'.

In practical terms, our alternate approach is really quite simple. If Belly-Up's senior debtholders expect that only $190 million of their $220 million in claims against the company will eventually be legally recognized (regardless of asset values), then Belly-Up's senior debt can be priced by setting the strike price of the underlying, written call at $190 million, rather than $220 million. To put this in terms of a 'dividend', we say that we are reducing the strike price by $30 million.

An investor should bear in mind when using this technique that it *is* a short cut. We find this approach appropriate when an investor is fairly confident of the extent to which her claim amount will not be recognized in full. However, if significant questions remain about how settlement claims will hold up against more junior claimants, an investor is well-advised to use the admittedly laborious, but more accurate, Cox and Rubinstein approach.

Settlement to Debtholders (cont'd)

Returning to Belly-Up, Inc., we will simplify matters by following the 'short cut' approach. In other words, when pricing our underlying options we account for the expected reductions in settlement amounts by making adjustments directly to the value of *K*.

Let us examine the effect of expected reductions in settlement amounts to senior debtholders first. It should be no surprise to find, as seen in Figure 16, that the value of common stock and junior debt rises, and the value of senior debt falls, when the 'full' settlement amount to which senior debtholders are entitled is reduced. We should note here that this graph is based on an assumption that these lower settlements will still be received 27 months hence, and that junior debtholders will suffer no reduction in their $180 million in claims.

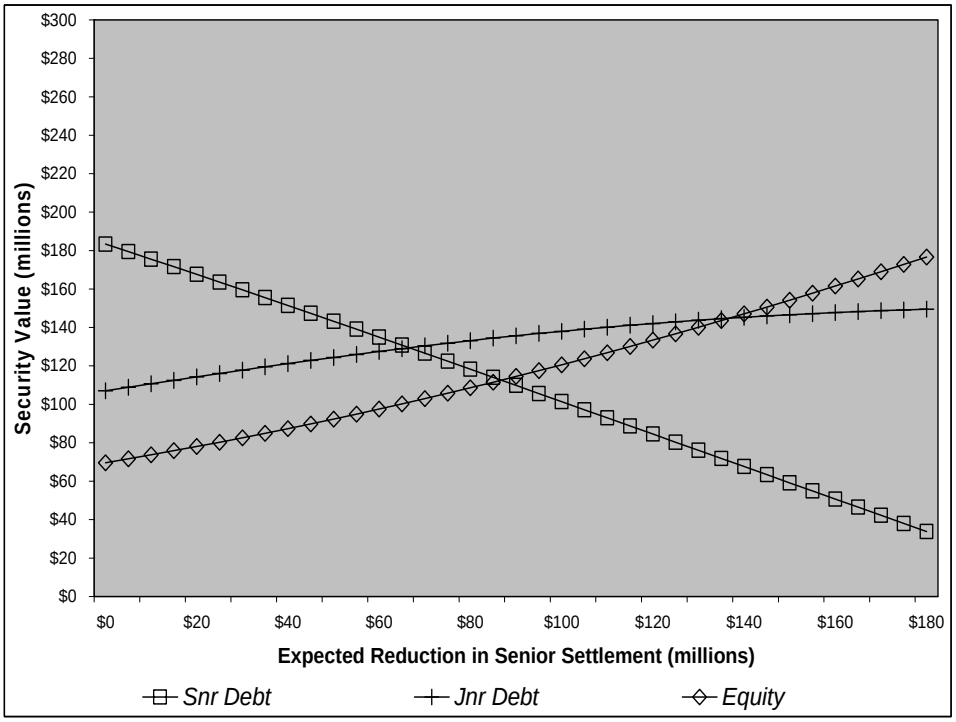

Figure 16 - Security value as a function of expected reductions in settlements to senior debtholders (assuming full, $180 million settlement to junior debtholders)

Settlement to Debtholders (cont'd)

Figure 17 shows security prices when senior debtholders expect to receive all of their $220 million in claims, but junior debtholders accede to or expect to receive less than their $180 million in

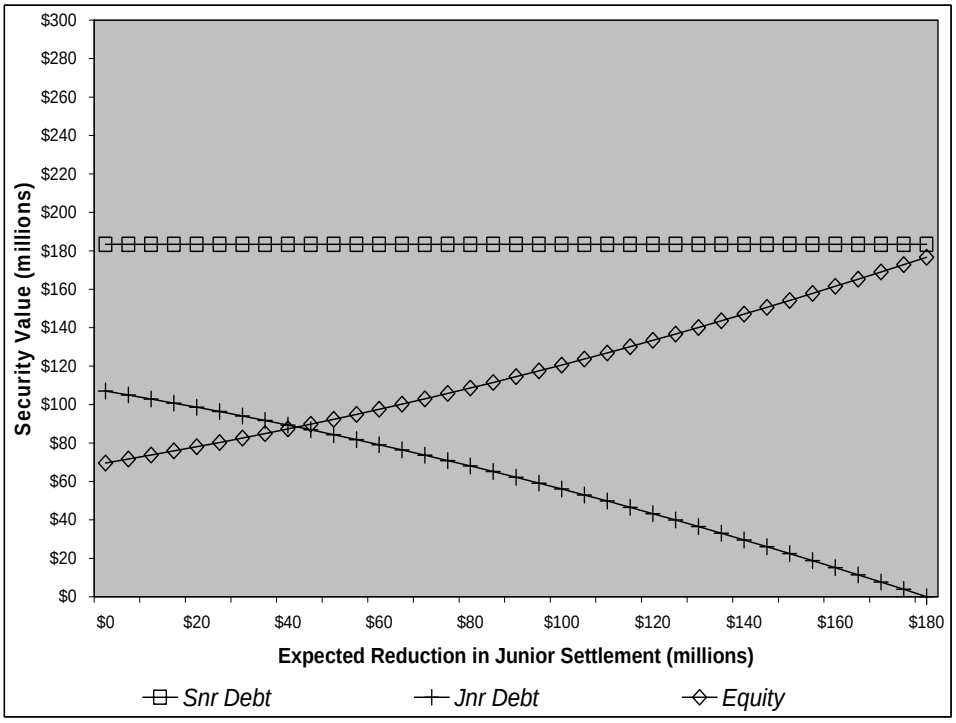

Figure 17 - Security value as a function of expected reductions in settlement to junior debtholders (assuming full, $220 million settlement to senior debtholders)

claims. As can be seen, equityholders reap all of the benefits when this occurs.

Finally, we convert both price-level diagrams to rate-of-return diagrams, which, in practice, will tend to be useful only when anticipating the effects of a cram-down. These rates of return are shown for reductions in ultimate settlement amounts of senior debt (see Figure 18), and junior debt (see Figure 19).

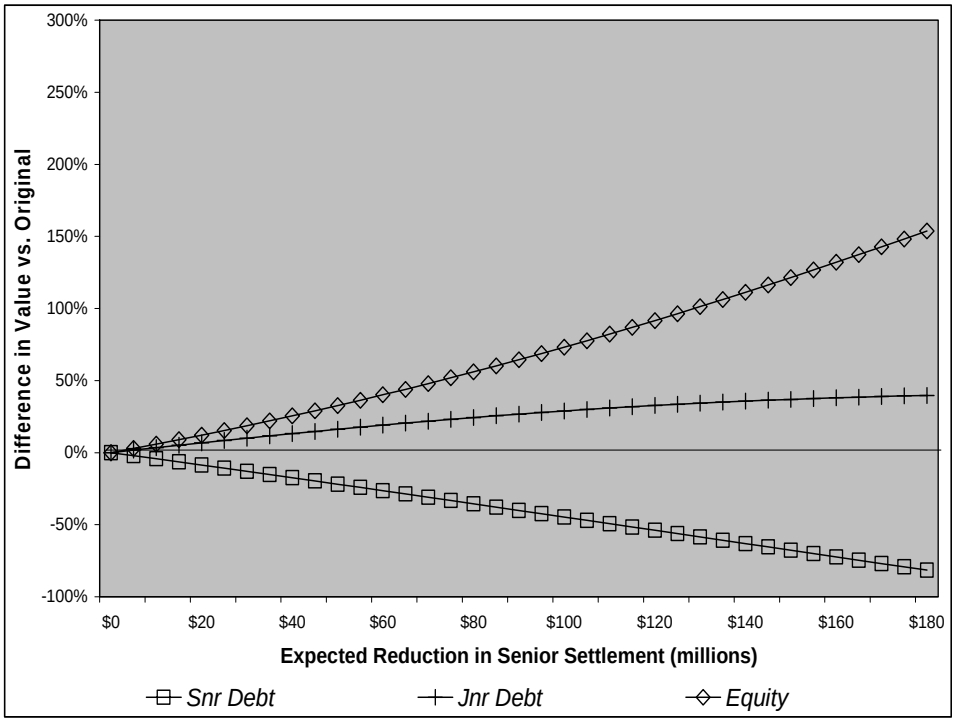

Figure 18 - Returns realized by investors as expected settlement to senior debtholders is reduced from $220 million

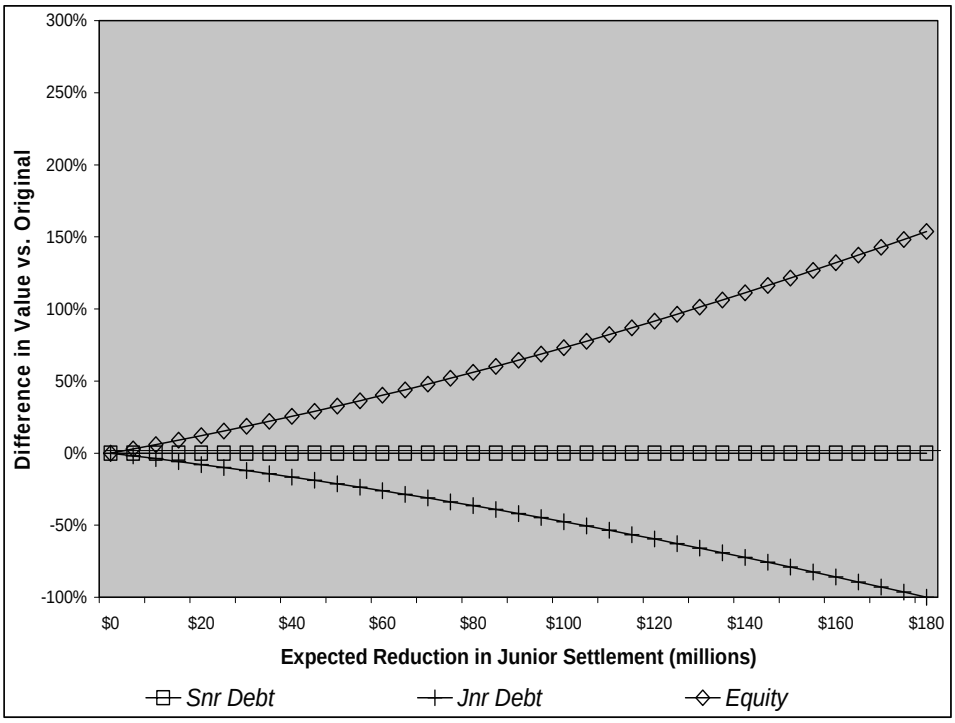

Figure 19 - Returns realized by investors through expected reductions in settlement to junior debtholders

Settlement to Debtholders (cont'd)

While our practice throughout this paper has been to look at the security price sensitivity to other relevant variables, the sensitivity of Belly-Up's security values to changes in anticipated settlement amounts will not be analyzed. This analysis is omitted because first-derivative, sensitivity equations are based on an assumption of small, continuous changes, yet in practice changes in anticipated settlements to bondholders are usually going to be large.

CONCLUSION

This concludes our illustration of how investors can utilize the Black-Scholes model to value Chapter 11 securities. We have reviewed each of the variables that determine call option values and have seen their respective influences on the value of Chapter 11 securities. While we have to be careful when drawing generalized conclusions about Chapter 11 securities from our specific example of Belly-Up, Inc., we are probably safe in saying the following:

- •Subordinated debt seems to be immune to interest rate levels. At least this will be the case when there are ample assets to repay senior debt, but are only 'just enough' assets to repay junior debt.

- The volatility of company assets plays an important role in determining security prices. This is especially true as the value of assets approaches the present value of outstanding debt.

- The expected length of time in bankruptcy proceeding affects Chapter 11 security prices not only because of the time value of money, but also because the total volatility of company assets is in part a function of time.

- As is well-known, asset values play a critical role in determining Chapter 11 security prices. However, the risk in holding Chapter 11 securities is often determined by the particular level of asset values. Specifically, when the value of company assets is close to the present value of outstanding debt, debt resembles an at-the-money call, and is therefore extremely volatile.

Ultimately, any investor who uses our approach to valuing Chapter 11 securities will want to analyze price behavior using the inputs specific to the firm under consideration. It is hoped that this paper will provide some guidance on what variables should be analyzed first. At the very least it is hoped that this paper has illustrated the dynamic analysis possible in applying the Black-Scholes model to Chapter 11 securities.